Developing MSA- and HSA-Style Questions

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Turn up the Heat:
Hot Ideas for Mathematics Education

What Types of Problems are on the MSA/HSA?

Selected Response (SR)

Student-Produced Response (SPR)

Brief Constructed Response (BCR)

Extended Constructed Response (ECR)

What does an MSA/HSA Problem Look Like?

The problem is in context.

Scored holistically on up to 5 rubric categories.

Measures only one or two indicators.

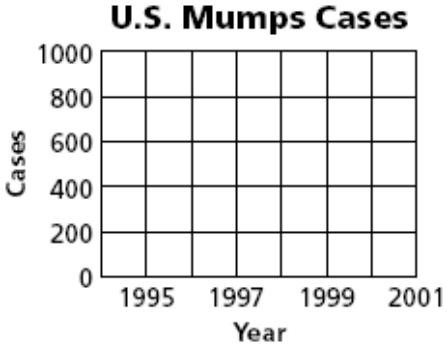
Typical Textbook Problem

DISEASE For Exercises 3–6, use the table that shows the number of cases of mumps in the United States for the years 1995 to 1999.

U.S. Mumps Cases					
Year	1995	1996	1997	1998	1999
Cases	906	751	683	666	387

Source: Centers for Disease Control and Prevention

3. Draw a scatter plot and determine what relationship, if any, exists in the data.



4. Draw a line of fit for the scatter plot.

5. Write the slope-intercept form of an equation for the line of fit.

6. Predict the number of cases in 2004.

Taken from Glencoe Algebra 1, © 2003 Resource materials

Typical MSA/HSA Problem

16 **BCR**

A tire company wants to determine how quickly the tread on its tires wears down with average use. Let x represent the number of months the tire was used. Let y represent the thickness of the tire tread, in millimeters. An equation for a line of best fit is shown below.

$$y = -\frac{5}{9}x + 20$$

Complete the following in the Answer Book:

- What is the slope of this line of best fit? What does the slope mean in the context of this problem?
- What is the y-intercept of this line of best fit? What does the y-intercept mean in the context of this problem?
- Tina will need to replace her new tires when they have 5 millimeters of tire tread left. According to the line of best fit, for how many months can Tina drive before she needs to replace her tires? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

Taken from the 2005 public release version of the Algebra/Data Analysis H.S.A.
Available at http://mdk12.org/assessments/high_school/look_like/algebra/intro.html

How are they similar?

- Both have a context.

- Both ask the students to use the line of best fit for a prediction.

How are they different?

- The MSA/HSA problem asks for the meaning of the slope and y-intercept.
- The textbook problem asks students to find the equation for the line of best fit.
- The MSA/HSA problem asks the student to explain his/her answer.

What is a Selected Response (SR) Item?

- These are multiple choice items.
- Work in a context as often as possible.
- It should take students a little more than a minute to answer an SR.

What is a Student-Produced Response (SPR) Item?

- These responses will require students to record their answers on a grid by shading in circles corresponding to the numbers in their answer
- Problem situation in context.
- Question includes label, so it is not needed in the answer
- No negative numbers or very large numbers (greater than 99,999) are possible
- It should take students about two minutes to solve

Start with a textbook problem

- Stem (start of the problem) of a few sentences, which is followed by a question

- Some problems will include a graphic (graph, model, etc.)

- No extra information

How many choices should we use?

- Four answer choices – ABCD – usually on separate lines

- ‘None of These’ option will not appear as an answer choice on the Maryland selected response items

What are distracters?

- Good distractors need to be clearly wrong (no ‘best’ choice – except for estimation)

- Wrong answers should be achieved by making only one error

How should choices be balanced?

- Either all four answers should be the same length or two short and two long

- Answers should appear in order – ascending or descending

Editing an SR Item Original 1

Sample 1
Original:

To rent a jetski at the shore, a customer must pay a $100 deposit on the jetski in addition to the $5 rental fee per hour. When the jetski is returned without damage, the customer gets half of the deposit back.

Which equation represents the total cost *(c)* over time *(t)* for jetskies returned without damage?

A. $c=5t+50$
B. $c=5t+100$
C. $c=50t+5$
D. $c=100t+5$

http://mdk12.org/assessments/high_school/developed/evolution_math.html

Editing an SR Item Edited 1

Revised:

Look at the sign below.

Which equation below represents returning a rowboat undamaged? Let *c* be the total cost in dollars and *t* be the time in hours.

A. $c=5t+100$
B. $c=500t$
C. $c=100t+5$
D. $c=5t$ *

Reason:

The term "jetski" was changed in this item for several reasons. Not all students will know what a "jetski" is, and some may become confused. "Jetski" is a licensed name for a commercially produced product the usage of which may require expensive permissions. The rental rate of $5 per hour is not realistic for a jetski. The sign graphic was added to decrease wordiness. Language was simplified to reduce wordiness. Return of only half of a deposit is not common, so a more realistic refund of the entire deposit was incorporated. Distractors were revised to match the revisions in the stem and were placed in a staggered pattern according to length.

http://mdk12.org/assessments/high_school/developed/evolution_math.html

Editing an SR Item Original 2

Sample 2
Original:

One hundred students at Heritage High School were sampled and asked their preference for a school mascot. The table below displays the results of the poll.

Mascot	Tiger	Lion	Dolphin	Hawk
Number of Votes	19	22	17	42

The total population of the school is 1550. Based on the survey, how many students at Heritage High School will vote for the mascot to be a lion?

A. 341
B. 22
C. 651
D. 42

http://mdk12.org/assessments/high_school/developed/evolution_math.html

Editing an SR Item Edited 2

Revised:

A random sample of 100 students at Heritage High School voted on a school mascot. The table below shows the results of the vote.

MASCOT VOTES

Mascot	Tiger	Lion	Dolphin	Hawk
Number of Votes	19	22	17	42

The total population of the school is 1,550. Predict how many High School want the mascot to be the lion.

A. 22
B. 42
C. 341 *
D. 651

Reason:

The language in this item was simplified to avoid wordliness. Terms (such as "vote") were standardized and concepts were simplified to avoid confusion. Distractors were stacked in numerical order.

http://mdk12.org/assessments/high_school/developed/evolution_math.html

What are the differences between SR's and SPR's?

- Selected Response items are limited to four answer choices for students to choose from. Student-Produced Responses have an unlimited number of answers.

- SPR's sometimes have a single correct answer, sometimes the correct answers fall within a limited range.

What is a constructed response item?

BCR

ECR

Brief Constructed Response (BCR)

A BCR is an open-ended question that provides students with the opportunity to generate and weave ideas into a short response. It can be used to assess:

- understanding of factual knowledge,
- a student's ability to synthesize ideas into an explanation,

Taken from MSDE resources

Brief Constructed Response (BCR) Continued

It can be used to assess:

- a student's ability to describe a model and identify its advantages and disadvantages,
- a student's ability to select evidence and support ideas, and
- a student's ability to analyze a graph or diagram.

Taken from MSDE resources

Back

Extended Constructed Response (ECR)

An ECR can be used to measure a student's ability to analyze and respond to complex situations. If gives the student the opportunity to generate an extended response to a question. The ECR can be used to assess the same items as a BCR.

Taken from MSDE resources

Back

Writing Constructed Response Items

- Start with a textbook ‘word problem.’
  - Context is harder to develop than skill. Select a context that has meaning for students.
  - Describe in three good sentences. Use simple but authentic vocabulary.
  - Should be context neutral (but you can do what you like with your students).

Writing Constructed Response Items

- Include necessary information.
  - Don’t put in extraneous information. Focus the student’s attention on the particular area of knowledge that they should use.
  - Information can be in graphic form, such as equation, table, graph, figure, diagram. Be sure to direct students to examine the graphic if you use a graphic. This can cut the number of words.

Writing Constructed Response Items

- Be clear and concise.
  - Tell students what they are to do and what is expected of them.
  - Tell students where they are to write their response and give them enough room on the paper to write their response.
  - Provide proper cueing to direct the student's thinking.

Writing Constructed Response Items

- Ask the question(s).
  - Use a bulleted list.
  - Here is where there is a difference between MSA and HSA:
    - MSA items have two parts. Part A is the content answer, part B is the process solution
    - HSA items have a single solution process and scored holistically

How Will You Use Each Rubric Category?

- Application
- Explanation
- Justification
- Connection
- Extension
- Representation
- Analysis

Application

- Refers to the appropriate concepts and strategies applied to solve the problem correctly. The solution is written in the context of the problem. The appropriate units are used.

- Scorers are looking at how the student applied mathematical concepts.

Application for Students

- Did you apply the strategies correctly? Did you get the correct answer? Did you show all of your work?

- Did you make sure that your answer is reasonable?

Explanation

- Refers to explaining the processes used to solve the problem.
- In words: using the language of mathematics
- In symbols: showing work step by step

Explanation for Students

- Did you communicate the answer to the problem and how you arrived at your answer?
- Did you tell why you solved the problem as you did?
- Would other people understand how you solved the problem by reading your answer?

Justification

- Refers to using definitions and properties to determine the validity of the solution.
- In words: Why is the solution appropriate for the situation?
- In symbols: Checking work step by step

Justification for Students

- Why did you do it?
- State the mathematical principles you used in solving the problem.
- Justification and explanation are inter-twined, but justification requires you to show your knowledge of concepts under-lying your explanation.

Connection

- Students connect to different topics within mathematics or to topics outside of mathematics.

- This connection can link different mathematical topics (such as fractions to proportions) or connect mathematics to other subjects (such as graphs to science)

Extension

- Again, students are ask to extend to other topics in mathematics or to topics outside of mathematics

- Extensions can involve changing values of given variables and examining what happens to the value of the answer

Representation

- The appropriate display of information.
- Graphs have a Title, Axes with consistent Intervals, appropriate Labels and Scales (T.A.I.L.S.)
- Data are graphed correctly.

Representation

- Lines and curves are labeled with their equation.
- Equations have variables defined.
- Tables are labeled correctly.

Analysis

- Refers to understanding the problem and determining the appropriate strategies or processes to solve the problem.
- When we analyze a problem, we break it into its components and make sure we understand what it is asking and what approach we need to take to solve it.

Analysis for Students

- Write down what it is asking you to do or what you could do to solve the problem.
- Write down the information given and what you need to find.
- Ask yourself: “Did I look at this problem as fully as I could?”

H.S.A. Style Guide

ethnic groups

- Hyphenate when used as an adjective, but not when used as a noun, such as the “African-American author” and “many Asian Americans have . . .”

figures

- Figures should be drawn to scale unless doing so will cue the students to the correct response.
- A note below the figure should accompany figures not drawn to scale.
  - “Note: The figure is not drawn to scale”

radicals

- Answer choices expressed in radical form should also be given as decimals (two places).
- A squiggly equal sign denotes the approximate equivalence.

H.S.A. Style Guide

constructed response

- MSA: Part a for the answer, part b for the process.
- HSA: Do **not** assign each question a part, such as:
  - Look at the ____ below.

    a.

    b.

    c.
- Use bulleted statements directly after the stem

[Again, remember the differences between MSA items and HSA items]

H.S.A. Style Guide

- **explain**
  - "Explain how you determined your answer. Use words, symbols, or both in your explanation."

- **justify**
  - "Use mathematics to justify your answer."

Levels of Cognitive Demand

Level 1 - Knowledge and Comprehension

recall, observe, know, identify, one-step problem solving

Level 2 - Application and Analysis

apply, multi-step problem solving, extend

Level 3 - Synthesis and Evaluation

generalize, predict, make conclusions, verify

How do we convert textbook problems into MSA/HSA problems?

Sample Textbook Items

- Grade 6: Properties of Quadrilaterals
- Grade 7: Probability
- Grade 8: Indirect Measurement
- Algebra 1: Patterns
- Algebra 1: Simulation

Summary Forward

Properties of Quadrilaterals Textbook Item

MSA Connection

Draw an example of each polygon. Mark any congruent sides, congruent angles, and right angles.

7. parallelogram **8.** pentagon **9.** rectangle **10.** decagon

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Taken from Glencoe Mathematics Applications and Connections, Course 1

Probability Textbook Item

MSA Connection

State the number of possible outcomes for each event.

3. Two dice are rolled.

5. You have a choice of 6 entrees and 4 desserts.

Return

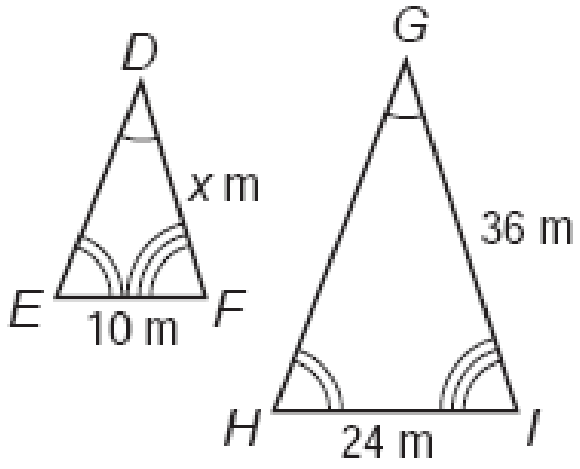
Taken from Glencoe Mathematics Applications and Connections, Course 3

Indirect Measurement Textbook Item

MSA Connection

In Exercises 1–10, the triangles are similar. Write a proportion to find each missing measure. Then find the value of x.

6.



Return

Taken from Glencoe Pre-Algebra © 2003 Resource materials

Pattern Textbook Item

HSA Connection

Find the next two items for each pattern. Then find the 19th figure in the pattern.

1. 

2. 

Return

Taken from Glencoe Algebra 1 © 2003 Resource materials

Simulation Textbook Item

HSA Connection

For Exercises 1–3, place 5 red, 4 yellow, and 7 green marbles in a box. Randomly draw two marbles from the box, record each color, and then return the marbles to the box. Repeat this procedure 50 times.

1. Based on your results, what is the experimental probability of selecting two yellow marbles?

2. Based on your results, what is the experimental probability of selecting a green marble and a yellow marble?

3. Compare your results to the theoretical probabilities.

4. Color blindness occurs in 4% of the male population. What could you use to simulate this situation?

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Taken from Glencoe Algebra 1 © 2003 Resource materials

Sample MSA/HSA Items

- Grade 6: Properties of Quadrilaterals SR
- Grade 7: Probability SPR
- Grade 8: Indirect Measurement BCR
- Algebra 1: Patterns ECR
- Algebra 1: Simulation BCR

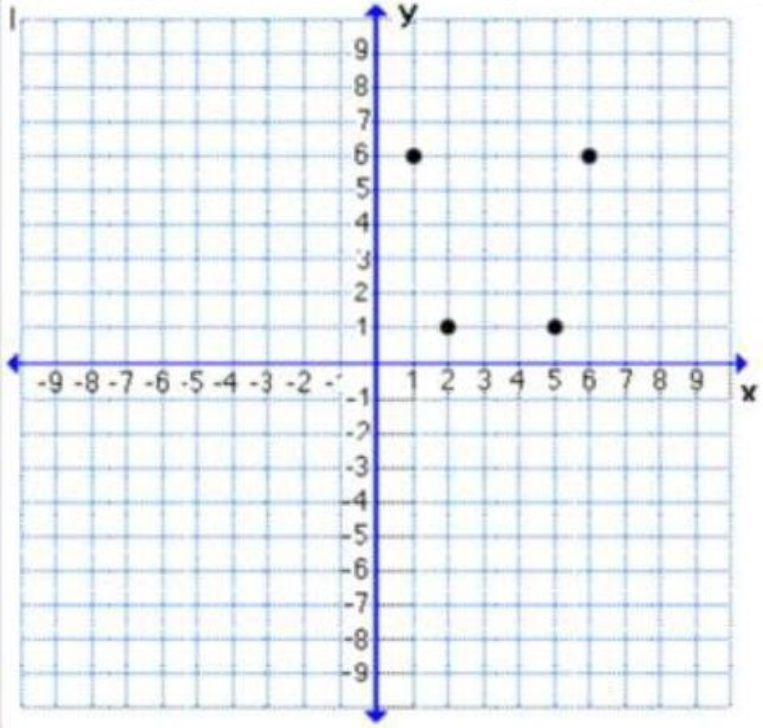
Grade 6: Quadrilateral SR

Textbook

Return

Selected Response Item

Maurice is given the coordinates of four points (1, 6), (6, 6), (5, 1), and (2, 1).



If he connects the points together, what quadrilateral is created?

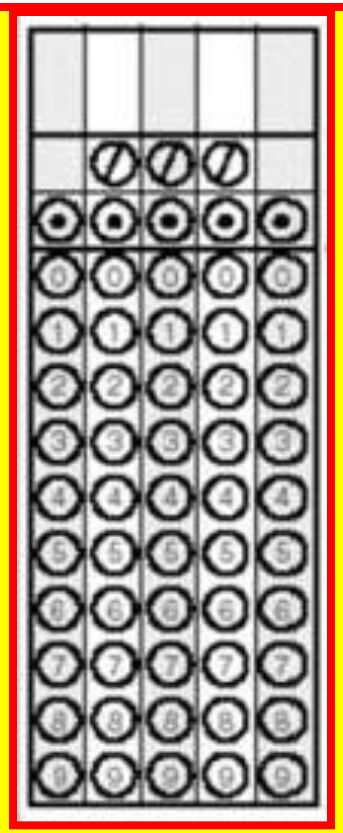
A. a rectangle
B. a square
C. a trapezoid
D. a rhombus

Taken from the Grade 6 VSC Toolkit

Grade 7: Probability SPR

Student Produced Response (SPR) Item

What is the total number of outcomes for picking a card from a group of 6 cards labeled A, B, C, D, E, and F, tossing a coin and rolling a number cube?



Textbook

Return

Taken from the Grade 7 VSC Toolkit

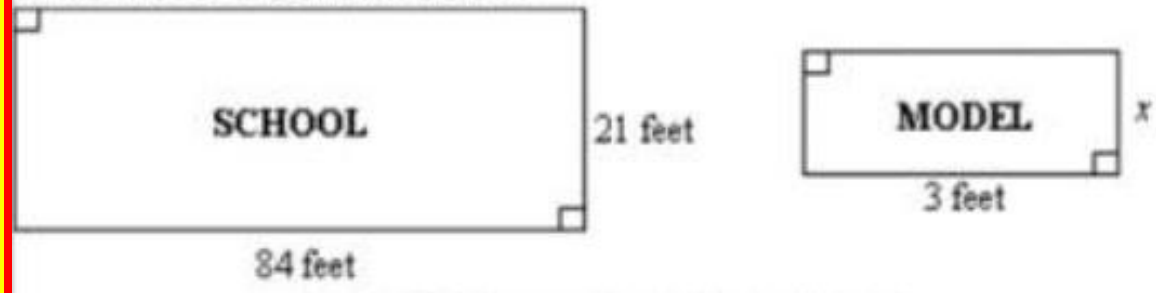
Grade 8: Indirect Measurement BCR

Textbook

Return

Extended Constructed Response (ECR) Item

Cedric is making a scale model of his school. The dimensions of his school and of his model are shown below.



Note: The figures are not drawn to scale.

Step A

What is the length, in feet, of side *x* of the model?

Step B

- Use what you know about similar polygons to justify why your value of side *x* is correct. Use words, numbers, and/or symbols in your justification.
- If Cedric changes the length of his model from 3 feet to 6 feet, explain how this change will affect the value of side *x*. Use words, numbers, and/or symbols in your explanation.

Taken from the Grade 8 VSC Toolkit

Algebra 1: Pattern ECR

12 **ECR**

Aisha used squares to make the pattern of figures below.

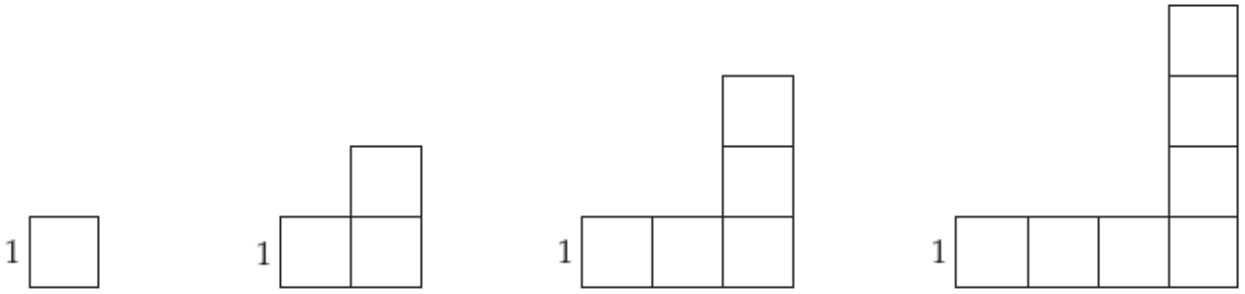


Figure 1 Figure 2 Figure 3 Figure 4

Complete the following in the Answer Book:

- Using the pattern, draw Figure 5 and Figure 6 in the Answer Book.
- Complete the table in the Answer Book to determine the perimeter of each figure.
- Write an expression that can be used to determine the perimeter of the *n*th figure in this pattern.
- If this pattern continues, which figure will have a perimeter of 140 centimeters? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

Textbook

Return

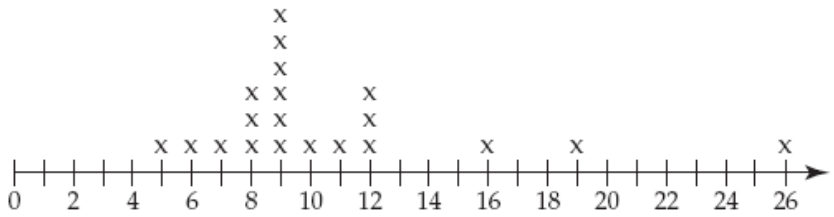
Taken from the Algebra 1 CLG Toolkit

Algebra 1: Simulation BCR

6
BCR

A cereal company awards a prize to anyone who collects all 5 different game pieces. Each game piece has an equal chance of being placed in a box of cereal. Each box of cereal contains 1 game piece. A class of 20 students conducted a simulation to see how many boxes of cereal must be purchased to collect all 5 different game pieces. Their results are shown in the table below.

SIMULATION RESULTS



Number of Boxes Needed to Collect All 5 Pieces
x = 1 Student

Complete the following in the Answer Book:

- What are the mean, median, and mode of the data?
- A box of cereal costs $2.80. Based on the simulation results, how much money must a consumer spend to collect all 5 different game pieces? Use measures of central tendency to justify your answer.
- One student in the class suggested that the data point of 26 should be ignored. Will ignoring this data point have a greater influence on the mean or the median? Use mathematics to justify your answer.

Textbook

Return

Taken from the Algebra 1 CLG Toolkit

Summary

Context is important

Students demonstrate skills through application

Summary

Solution Process

Context to Mathematical Model to Application (bringing back to the context)

Summary

The difference between a BCR and an ECR is the amount of time a student needs to complete the item:

BCR - 8 minutes

ECR - 15 minutes

Summary

Be sure to consider what you expect as a student response:

Application

Explanation/Justification

Connection/Extension

Representation

Analysis

Summary

Many textbook problems can be converted into MSA/HSA-style problems, but not all can be.

Summary

Clarity and simplicity is important. If a context is too complex, it may not be appropriate for an item.

Sometimes a graphic can replace wordy text.